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The Size, Number,
and Mineral-Carrying Efficacy
of Bubbles in Flotation

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The Size, Number,
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of Bubbles in Flotation

The Ideal Flotation Machine -

The ideal flotation machine may be defined as one that: 1) will recover unit weight of mineral in minimum of time; and 2) will make this unit recovery a) in minimum cell volume, b) in minimum floor space, c) with minimum expenditure of horsepower, and d) at minimum maintenance cost. The economic factors are:

1. Rate of recovery;
2. Grade of concentrate;
3. Power consumption; and
4. Maintenance or operating cost.

The first cost of equipment, within reasonable limits, is a minor factor.

It will be obvious from the experimental data to follow, that no machine, embodying fixed and inflexible design features, can qualify as the "ideal machine" on all types of ore. Experience has amply proved this. The modern flotation machine must, of necessity, therefore, embody mechanisms to enable the operator to adjust his factors to meet the particular conditions at hand.

The Rate, Degree, and Uniformity of Pulp Aeration -

The "rate of aeration" in flotation is here defined as the volume of air escaping from the pulp in unit time from unit cell area. The units may be cc./min./sq.in., or cu.ft./min./sq.ft. In this report, rate of aeration is recorded in cc./min./sq.in.

The "degree of aeration" is defined as the percentage of air, by volume, contained in the pulp. Degree of aeration is perhaps best stated in terms of cubic centimeters per liter, or in per cent of pulp volume. Degree of aeration is a factor affecting the specific gravity of the pulp -- it can be as much as 20 to 25 per cent.

Rate of aeration and degree of aeration are not necessarily one and the same thing. When, for any reason, the bubbles are very small -- their rising velocity low -- the degree of aeration may get very high, but on the other hand, the rate of aeration very low. Small bubbles, high degree of aeration and low rate of aeration is a combination that can go so far as to almost completely defeat flotation. This experience was encountered particularly in working with magnesite ore.

As will be shown in some later experiments, all frothers materially affect bubble size, and this is one, no doubt important, reason why in many operations careful control of the frother is so vital. It would seem to

follow that in coarse-sand flotation, particularly, the degree of aeration ought not be allowed to get too high. Bubbles -- at least a sufficient number of them -- must be large enough to dominate the behavior and disposition of the large, heavy mineral particles.

Bubbles, of any given size, when formed at a constant rate, have a definite "rate of escape" from a flotation pulp, and this rate is determined by many factors. Some are: 1) degree of aeration, 2) pulp specific gravity, 3) pulp depth, 4) mineral load on the bubbles, etc. For any given bubble size, "fewer" bubbles escape faster than many bubbles. Few bubbles escape under "free settling" conditions, while many bubbles (high degree of aeration) escape at a slower rate under "hindered settling" conditions.

The pulp, however, is denser for the condition of few bubbles than for the condition of many bubbles, and it therefore follows that "free rising" bubbles are smaller than the "hindered rising" bubbles. Mathematically, one might go to considerable length in an attempt to arrive at the net result, on rate of rise, of the factors of pulp density (bubble column density) and bubble size. One tends to nullify the other.

Bubbles are larger and escape at a higher rate in shallow cells than in deep cells.

Bubbles, because they are larger, escape faster from a pulp of low specific gravity than from a pulp of high specific gravity. There is a compensating factor here, however, because of the greater bubble buoyancy in the higher specific gravity pulp. Herein, no doubt, is one plausible explanation for the inability of the operator, in many instances, to obtain equal operating and metallurgical results with small and large cells on identical feeds, and why it is not always possible to duplicate laboratory (shallow cell) results in large (deep) commercial cells.

The rate of escape of a bubble is also influenced by the specific gravity of the mineral to be floated and by the extent to which the bubbles are loaded with mineral.

The Rate of Recovery and Related Factors -

An ore is said to be slow-, medium-, or fast-floating, depending upon the treatment time (in the cell) required for economic recovery. A slow-floating ore, as gauged by experience, requires up to 20 minutes -- a fast-floating ore, as little as 5 minutes. This classification is wholly unscientific, but is useful in estimating the number of cells needed for any particular job.

Some of the factors that govern the category into which an ore falls, with respect to rate of floating, are:

1. The native (or induced*) flotability of the mineral;
2. The mineral specific gravity factor;

* Meaning the flotability conveyed upon a mineral by use of standard, well-known collectors.

3. The percentage content of minerals to be floated;
4. The fineness of the mineral-sand to be floated;
5. The rate, degree and uniformity of pulp aeration;
6. The specific gravity of the pulp;
7. Cell depth;
8. The nature of pulp circuits and "pulp drafts".

Critical Particle Size and Critical Number -

"Critical-size particle" is defined as that particle which gives a mineral load on the bubble when the bubble is completely covered with particles, just equal to the buoyancy force of the bubble.

Since the surface (loading space) of a sphere (bubble) varies directly as the radius squared, and the volume (lifting capacity) as the radius cubed, it is obvious that the "critical particle" is a function of bubble size. It is also obvious that for any given size of bubble, that critical particle is a function of the specific gravity of the mineral to be floated. Particle shape also is a factor, but in this discussion only the cubic form is considered.

It is of interest to know something of the approximate magnitude of the critical particle, as defined above. In Table I. are figures resulting from calculations for the minerals galena (sp.gr. 7.5) and magnesite (sp.gr. 3.01) for four assumed bubble sizes. In these calculations, the particles are assumed to be cubic in form, and no account is taken of the volume not occupied by the particles due to bubble surface curvature and/or to imperfect particle-fitting.

TABLE I.

Critical Particle Size and Variable Bubble Size
For Galena and Magnesite

Bubble Size, Cm.	Radius Cubed	Surface Sq. Cm.	Volume CC.	Critical Particle Size	
				Galena: Sp.gr. 7.5	Magnesite: Sp.gr. 3.01
				MM.	MM.
0.05	0.0000156	0.00785	0.0000655	0.0111	0.0277
0.10	0.000125	0.0315	0.000523	0.0222	0.0554
0.15	0.000423	0.0705	0.00177	0.0353	0.0830
0.20	0.001	0.1256	0.00418	0.0444	0.111

The data are arrived at by use of the formula:

$$4\pi r^2 d s = w$$

where r = radius of the bubble in cm.

d = depth of mineral layer = "critical particle" in cm.

s = specific gravity of mineral to be floated

and w = weight of mineral = buoyancy force of the bubble.

The "critical number" N , is defined as the number of critical particles carried by the bubble when the bubble is fully loaded. It is arrived at by the formula

$$N = 4\pi r^2 + d^2$$

where the symbols have the values stated above.

It is of interest to note that the critical particle size is four times as great for a .20 cm. bubble as for a .05 cm. bubble.

Considerations of this character would seem to indicate that the size of bubble might well be a most significant factor and one that is no doubt importantly related to such other factors as fineness of grinding and rate of flotation.

The experiments to follow are designed to bear on these factors.

Rate of Recovery and the Fineness of Grind Factor -

Grinding is required to effect mineral liberation and to reduce mineral to flotation particle-size. The operation always results in what is known as "overgrinding." There is no way, however, completely to prevent overgrinding, and in practically all flotation feeds there are present many "sub-optimum" size particles. Some particles are so small that, for reasons not yet known to scientists, collector reagents fail to confer upon them bubble-attachment properties. The number of such sub-optimum particles may be held to a minimum by a grinding practice employing high circulating loads and/or by use of a flotation cell in the grinding-classifier circuit.

If a given weight of massive floatable-mineral were reduced to a "sand" of uniform grain size and shape (physically impossible), there would undoubtedly be a specific size -- "optimum size" -- at which the given weight could be floated in minimum time. As far as the writers know, the optimum size, as above defined, has not been determined and is not known for any mineral. Such a size (optimum size), being determined by many factors, actually may not exist.

Using massive galena of the "cube" variety, experiments were devised to produce data bearing on the question of particle size in relation to the rate of flotation. For these tests, artificial mixtures of galena (50 gms.) and magnesite (450 gms.) were prepared for head samples. The galena was sized with the $\sqrt{2}$ sieves so that only one size of particle was present in each head

sample. The magnesite, serving as the gangue mineral, was pulverized so that all passed a 48 mesh sieve.

The technique employed was as follows:

- 1) The volume of water was carefully measured so that the volume of the pulp was the same for each test.
- 2) A guide was fastened on the froth-skimming paddles so that the same depth of froth was removed with each raking.
- 3) The frothing time was measured from when the air was turned on until the last froth was removed for each concentrate.

The following reagents and conditioning procedure were used in each test in this series:

Ore	500 gm. (galena - 50, and magnesite - 450)
Water	2200 cc.
Na ₂ CO ₃	0.25 gm. (1 lb./ton)
Z-5	0.02 gm. (0.08 lb./ton)
Conditioning time	3 minutes
Pine oil	5 dr. (0.18 lb./ton)

Four separate concentrates were collected and weighed in each test, but since the bulk of the galena was contained in the first one (especially in the finer sizes), it was used as the basis of comparison. Since the products were composed of only two minerals, approximate assays were made by determining the specific gravities of the sand mixtures.

The metallurgical results of this series of tests are given in Table II.

TABLE II.

The Rate of Flotation and the Particle Size Factor,
Galena - Sized

Test No.	Galena, Size of Particles	Concentrate		Galena in Conct. Wt., Gms.	Time of Frothing	Rate of Flotation, Gms. Galena Per Minute
		Wt., Gms.	Galena in Conct. %			
1	28/35	2.39	56.8	1.36	16.0	5.1
2	35/48	6.68	74.6	4.99	13.5	22.1
3	48/65	19.36	87.5	16.9	15.5	65.5
4	65/100	17.26	86.0	14.8	10.5	84.6
5	100/150	28.21	85.7	24.2	10.0	145.0
6	150/200	33.73	86.7	29.2	8.0	219.0
7	200/270	37.71	87.2	32.8	8.7	227.
8	270/400	40.47	85.0	34.3	8.1	254.
9	-400	31.46	80.5	25.3	8.0	190.

A study of the data of Table II discloses:

1) That the largest particle floated is .508 mm. -- the galena grains are uniformly cubic in form.

2) That the rate of flotation (grams per minute) increases as the particles get smaller -- in the size range tested -- down to and including the 400-mesh (.038 mm.) size. In the light of the fact that the 270/400 mesh particle is of critical particle magnitude (see Table I), it is probable that the flotation rate on this size particle is near the maximum.

3) That the grade of concentrate is nearly constant -- in the size range tested -- for all sizes between 48 and 400 mesh.

In another test, using galena and magnesite to make up the pulp, the galena content in the pulp consisted of a mixture of sizes -- 5.55 grams of each of the following: 28/35; 35/48; 48/65; 65/100; 100/150; 150/200; 200/270; 270/325; and -325 mesh. The conditions obtaining in this test were as stated above. The result is compiled in Table III.

TABLE III.

The Rate of Flotation and the Particle Size Factor --

Galena, Mixed Sizes

Galena		Weight of Concentrate Grams	Per cent Galena in Concentrate	Weight of Galena in Concentrate	Grams of Galena Floated per Min.
Mesh	Mm.				
20/28	0.711				
28/35	0.503	0.24	56.4	0.135	1.25
35/48	0.356	0.54	74.2	0.400	3.70
48/65	0.251	0.92	88.0	0.810	7.50
65/100	0.178	1.09	79.8	0.870	8.01
100/150	0.125	1.52	87.2	1.150	10.55
150/200	0.089	1.65	87.2	1.44	13.40
200/270	0.065	2.00	87.5	1.750	16.20
270/325	0.048	2.10	87.0	1.830	17.00
-325	-0.002*	2.00	85.0	1.700	15.70
				TOTAL -	93.31

* Assumption

Note: Frothing time - 6.5 sec.

From Table III it may be noted:

1) That, in mixed particle-size flotation, the bubbles carry a greater weight of particles smaller than 150 mesh than of grains coarser than this size -- this is consistent with the data of Table II, and

2) The overall rate of flotation (grams per minute of all sizes), is considerably less than the best rate of flotation on sized particles as seen in Table II.

In similar tests with the mineral sphalerite (data to appear later), the sizing and flotation are carried into the sub-sieve range. Questions that naturally arise at this time are considered then.

Estimate of the Number of Grains Floated per Bubble -

A .711 mm. galena particle weighs ($[0.711]^3 \times 7.5 =$) .00270 grams; therefore, the minimum bubble capable of lifting this particle must needs have a buoyancy force of .0027 grams. The volume of a sphere is equal to $\frac{4}{3} \pi \left(\frac{d}{2}\right)^3$; therefore, the diameter of the bubble of buoyancy .0027 grams, is 1.74 mm.

It therefore follows that some, if not all, of the bubbles in the flotation tests above had diameters of at least 1.74 mm. Observations (by a technique to be mentioned later) lead to the belief that the bubbles -- and they appeared to be uniform in size -- are of the magnitude 1.74 mm. Bubbles formed in water are very much larger than this, but the pine oil, in the amount used, promotes an aeration consisting of very much finer bubbles.

Continuing with the analysis. A gram of galena occupies a volume of 0.133 cubic centimeters. This is the volume of a galena cube .5105 cm. The volume of air used by the flotation machine was measured to be .25 cu.ft./min., equal to 7080 cu. cm.

Taking the size of bubble formed as 1.74 mm., the number of bubbles made by the machine is $(7080 \div 0.0027 =)$ 2,620,000 per minute.

We are now in position to calculate bubble load in galena particles for each size.

These calculations are summarized in Tables IV and V.

TABLE IV.

Number of Galena Particles of Various Sizes Floated per Bubble

Galena Size of Particles		Number of Particles per Gram	Grams Galena Floated per Minute	Particles Floated per Minute	Particles Floated per Bubble
Mesh	MM.				
14/20	1.00	133			
20/28	0.711	371			
28/35	0.503	1,048	5.1	5,350	0.00204
35/48	0.356	2,955	22.1	65,200	0.0249
48/65	0.251	8,432	65.5	552,000	0.210
65/100	0.178	23,640	84.6	2,000,000	0.764
100/150	0.125	68,260	145.0	9,900,000	3.77
150/200	0.089	189,100	219.0	41,400,000	15.80
200/270	0.063	533,200	227.0	121,000,000	46.30
270/400	0.045	1,463,000	254.0	372,000,000	142.00
-400	0.002*	16,660,000,000 or 16.6×10^9	190.0	3,140,000,000,000 or 3.14×10^{12}	1,195,000.00

* Assumption

Note: Particles floated per bubble calculated from 2,620,000 bubbles per minute

TABLE V.

Number of Particles Floated per Bubble in Feed of Mixed Sizes

Galena Size of Particles		Number of Particles per Gram	Grams Galena Floated per Minute	Particles Floated per Minute	Particles Floated per Bubble
Mesh	MM.				
14/20	1.00	183			
20/28	0.711	371			
28/35	0.503	1,048	1.25	1,310	0.00050
35/48	0.356	2,955	3.70	10,900	0.00416
48/65	0.251	8,432	7.50	63,200	0.0241
65/100	0.178	23,640	8.01	189,000	0.072
100/150	0.125	68,260	10.55	720,000	0.275
150/200	0.089	189,100	13.40	2,530,000	0.965
200/270	0.063	533,200	16.20	8,640,000	3.50
270/325	0.048	1,205,600	17.00	20,500,000	7.84
-525	0.002*	16,660,000,000 or 16.6×10^9	15.70	261,000,000,000	99,700.00

* Assumption

The surface of a 1.74 mm. bubble is $(4\pi[0.87]^2) = .094$ sq. cm. The buoyancy force of the bubble is .00270 gm., equal to $\frac{.0027}{7.5} = .00036$ cubic cm. of galena. Now the thickness of a spherical shell of galena whose volume is .00036 cubic cm., and whose weight is .0027 gm., and whose inside diameter is 1.74 mm., is found as follows:

$$\begin{aligned} .094 \text{ sq. cm.} \times X \times 7.5 &= .0027 \\ .705X &= .0027 \\ X &= 0.0038 \text{ cm.} = 0.038 \text{ mm.} \end{aligned}$$

Thus, a 1.74 mm. bubble fully packed with mineral (galena) particles will carry the maximum load when the particles are .038 mm (400 mesh) in size. This is the critical particle size for galena for this size bubble.

The number of .038 mm. particles on the bubble when completely loaded is $(0.094 \div [0.0038]^3) = 6500$. This is the critical number.

Now it is of course obvious that a bubble fully loaded with a weight of mineral just equal to its buoyancy force, is not physically capable of lifting and depositing its mineral load in the surface froth.

If a fractional bubble-load is assumed, this fractional load may be attained in two ways: 1) complete surface coverage by "sub-critical" size grains, or 2) fractional surface coverage by critical size grains. In practice, unquestionably, bubble coverage is fractional and the load comprised of variable size particles.

Bubble Efficiency -- for Galena -

Table VI is a compilation of calculations designed to show "order of magnitude" figures on the efficiency of bubbles in flotation. The bubble size was taken at 1.74 mm. and the calculations are for galena. The feed pulps in the six tests from which these data were obtained, contained uniform (sieve sized) galena particles.

Column eight (8) of this table, gives the critical numbers for each particle size; column seven (7) the estimated number of particles floated per bubble; and column nine (9) the per cent of the critical number, or per cent efficiency.

Most certainly, high accuracy cannot be claimed for these data, but it is probable that the error is not greater than, say, 100 per cent. Allowing even for such high discrepancy, the data even so are instructive.

With particle-size and bubble-size such important factors, it is becoming clear why such factors as 1) pulp depth, and 2) kind and quantity of frothing agent, have such large bearing on metallurgical results and all-round machine performance.

TABLE VI.

Bubble Efficiency, The Mineral Galena

Assumed Bubble Size, 1.74 MM.

Galena		Grams Floated Per Minute	Particles		Particles Floated		Possible Number Particles Per Bubble	Effic- iency %
Mesh	MM.		Per Gram	Wt. Per Particle	Per Minute	Per Bubble		
1	2	3	4	5	6	7	8	9
28/35	0.503	5.1	1,048	0.000955	5,340	0.00204	2.83	0.072
35/48	0.356	22.1	2,955	0.000339	65,200	0.0249	7.95	0.313
48/65	0.251	65.5	8,432	0.000119	552,000	0.211	22.8	0.925
65/100	0.178	84.6	23,640	0.0000423	2,000,000	0.765	63.7	1.20
100/150	0.125	145.0	68,260	0.0000146	9,870,000	3.77	184.0	2.05
150/200	0.089	219.0	189,100	0.0000053	41,400,000	15.8	510.0	3.10
200/270	0.063	227.0	533,200	0.0000019	121,000,000	46.2	1420.0	3.25
270/400	0.045	254.0	1,463,000	0.00000068	372,000,000	142.0	3960.0	3.59
-400	0.002*	190.0	16.6×10^9	6.02×10^{-11}	3.15×10^{12}	1.205×10^6	4.5×10^7	2.68

* Assumption

Rate of Recovery in Relation to Particle Size Factor -- Sphalerite --

It was thought that experiments, similar to those outlined above for galena, but using sphalerite would yield additional interesting data on the question of rate of recovery in relation to particle size. Sphalerite has a lower specific gravity, different fracture and different flotative characteristics than galena.

The data for sphalerite are presented in Table VII. These data result from eleven (11) individual tests in which the variable was particle size. In these tests, as with galena, artificial mixtures of sphalerite (50 gms.) and magnesite (450 gms.) were used. The magnesite was ground to -48 mesh for all tests. The frothing time was measured from the moment the air was turned on until the fifth raking was removed for each concentrate.

Reagents and conditioning were as follows:

Ore - 500 gm.
 Water - 2275 cc.
 Na₂CO₃ - 0.25 (1.0 lb./ton) conditioned for 1 min.
 CuSO₄ - 0.25 (1.0 lb./ton)
 Z-5 - 0.02 (0.08 lb./ton) conditioned 3 min.
 Pine oil - 5 dr. (0.18 lb./ton)

TABLE VII.

The Rate of Flotation and the Particle Size Factor, Sphalerite

Test No.	Particle Size (Mesh)	Wt. of No.1 Conct. (Gms)	% ZnS in Conct.	Wt. ZnS in Conct.	Frothing Time, Seconds	Flotation Rate, Grams ZnS/Min.
101	28/35	23.47	85.7	20.1	10	120.6
102	35/48	29.69	74.7?	22.2	9.6	139
103	48/65	33.40	81.0	27.1	8.2	198
104	65/100	43.78	84.3	36.9	8.4	264?
105	100/150	36.46	87.0	31.8	7.8	245
106	150/200	39.68	84.3	33.5	8.2	245
107	200/270	37.50	91.5	34.3	8.0	257
108	270/400	39.30	95.3?	37.4	8.0	281
110	400/800	33.29	85.9	28.6	7.8	220
111	800/1600	22.89	71.2	16.3	8.0	122
112	-1600	15.22	37.5?	5.7	7.4	46

As will be observed from the tabulated data (Table VII), the sizing* was carried into the sub-sieve range.

In these tests with sphalerite, the same volume of air was consumed by the test machine and it is assumed, for purpose of calculations, that the size and number of bubbles produced was the same as for the galena tests -- estimated at 2,620,000., 1.74 mm. bubbles per minute.

The purpose in carrying the study, in this case, into the sub-sieve range, was to determine the particle size at which the rate of flotation reached a maximum. It is observed from the data that this maximum flotation rate occurs at a size near 400 mesh (38 microns). It is assumed, of course, that there was sufficient xanthate to condition all sizes of particle -- although it might be argued that the falling off in rate of recovery in the sub-sieve range is due to insufficient xanthate. This, however, is not believed to be the case.

* Sizing was done by the sedimentation method, using large beakers, and the particle-size range in each product was estimated microscopically.

Bubble Efficiency for the Mineral Sphalerite -

In Table VIII are compiled some bubble-efficiency data, resulting from calculations similar to those employed in making up Table VI.

TABLE VIII.

Bubble Efficiency - The Mineral Sphalerite
 Size of Bubble, Estimated: 1.74 mm.
 Number of Bubbles per Minute: 2,620,000
 Surface of Bubble: 0.094 sq. cm.
 Buoyancy of Bubble: 0.0027 gm.

Sphalerite		Grams Floated Per Minute	Particles		Particles Floated		Possible No. of Particles Per Bubble	Bubble Efficiency %
Mesh	MM.		Number Per Gram X 10 ³	Wt. per Particle Gms X 10 ⁻³	Per Minute X 10 ³	Per Bubble		
28/35	0.503	120	1.97	0.507	236	0.090	5.3	1.7
35/48	0.356	139	5.56	0.179	774	0.295	15.1	1.95
48/65	0.251	198	15.8	0.0633	3,130	1.195	42.7	2.80
65/100	0.178	264	44.2	0.0226	11,650	4.45	120.0	3.71
100/150	0.125	245	128.0	0.0078	31,400	12.0	346.0	3.47
150/200	0.089	245	355.0	0.0028	87,000	33.2	965.0	3.44
200/270	0.063	257	1,000.0	0.001	257,000	98.1	2,700.0	3.63
270/400	0.045	281	2,740.0	0.00036	770,000	294.0	7,500.0	3.92
400/800	0.028	220	11,350.0	0.000088	2,490,000	951.0	30,700.0	3.10
800/1600	0.014	122	91,000.0	0.000011	11,200,000	4280.0	245,000.0	1.75
-1600		46						

It is interesting to note that the bubble efficiency -- roughly 3.5 per cent -- is fairly constant throughout the size range .177 to .045 mm., and further, that this efficiency figure for sphalerite is of the order of magnitude of that for galena.

It should be understood that these efficiency figures are for specific mineral concentrations and for the initial few seconds of frothing. The efficiency approaches zero as the test continues to the end.

The Critical Size and Number for Sphalerite -

The surface of a 1.74 mm. bubble is .094 sq. cm. The buoyancy force is .00270 gm., equal to .0007 cc. of sphalerite. Now the thickness of a spherical shell of sphalerite whose volume is 0.0007 cubic centimeters, and whose weight is .0027 gram, and whose inside diameter is 1.74 millimeter, is:

$$.094 \times X \times 4.00 = .0027$$

$$.376X = .0027$$

$$X = 0.0072 \text{ cm.} = 0.072 \text{ mm.}$$

Thus, a 1.74 mm. bubble, fully packed with sphalerite particles, will carry the maximum load when the particles are .072 mm. (about 200 mesh). This is the critical particle size for sphalerite for this size bubble. The number of .072 mm. particles on the bubble, when completely loaded, is $(0.094 \div (0.0072)^2) = 1810$. This is the critical number for sphalerite for a 1.74 mm. bubble.

The bubble efficiency at this size is about 3.5%.

Rate of Recovery and the Percentage-of-Mineral-Content Factor -

It is probably correct to state that ores, such as, for example, lead ores, which are similar in all respects -- excepting in mineral content -- require treatment times (cell-volumes) proportional to the mineral content of the respective ore. A 10 per cent lead ore requires longer treatment (greater number of cells) than a 5% lead ore. It is not known, however, if the time ratio is 2:1 or less. It is well known in practice that, all other factors being equal, the percentage recovery is a function of mineral content in the feed. In overall recovery -- and grade of concentrate invariably is better for the higher than for the lower-grade ore -- the lower-grade ore, however, always makes the lower-grade tailing.

Employing technique and test conditions already explained, in a series of experiments using -150+200 mesh galena, the mineral concentration in the pulp was made the variable. The flotation results are presented in Table IX.

TABLE IX.

Rate of Flotation and the Mineral-Concentration Factor -

-150+200 Mesh Galena

Test No.	Galena in Feed, %	Conct. Wt., Gms.	Galena in Conct. %	Galena in Conct. Wt.	Frothing Time Seconds	Rate of Flotation, Gms. Galena Min.
1	5.0	22.46	70.2	15.75	6.6	143
2	10.0	33.73	86.7	29.2	8.0	219
3	15.0	37.32	87.1	32.5	7.0	279
4	20.0	50.63	90.6	45.8	6.5	423

The above data suggest several novel means which might be resorted to in improving flotation economy.

The several factors that are practically constant in these tests are: 1) the depth of cell; 2) the amount of frother used, and in turn, the size of bubble made; 3) the number of bubbles formed in unit time; and 4) the time of residence of the bubbles in the pulp. The only variable -- the one under study -- is the mineral concentration.

Flotation Economy and the Cell-Depth Factor -

It is now clear that bubble efficiency in terms of the theoretically possible mineral load is extremely low, perhaps much less than one per cent. The efficiency of the bubble would appear, however, other factors being constant, to be a function of 1) mineral concentration c , and 2) the time t the bubble is in the pulp. In other words, the number of mineral particles N brought to the pulp surface, and deposited in the froth, per bubble, might be expressed mathematically as

$$N = Ktc$$

where K is a constant and includes such factors as the composition, specific gravity and particle shape of the mineral.

Now t , among other things, is a function of pulp (cell) depth and c , for any given mineral, ore-grade and grind, is a function of pulp consistency.

In practice, pulp to be floated, is fed to a flotation machine comprising many cells in series.

The cells are normally uniform in size (volume) and depth, and in the manner of their operation. The flotation feed is received either direct from the grinding-mill classifier or from a thickener. The consistency of the classifier overflow is largely controlled by the limitations of the grind, but if a thickener is employed the flotation feed, as to consistency, is subject to some control.

Now since t in the above equation is directly determined by the depth h of the flotation cell, we may write the equation:

$$N = Khc$$

In practice, h varies from two feet to about four feet.

Let us assume that N , for any given volume of c is a maximum when $h = h_N$. It is then obvious that if $h < h_N$, the bubble will carry less than a full mineral load into the froth and also that if $h > h_N$, the bubble will acquire its full load before reaching the pulp froth. Perhaps, if the ideal pulp depth cannot be hit upon, the condition of $h < h_N$ is the more desirable condition economically, owing to the high rate of power consumption with pulp depth. The second, third, and successive cells all will, in most practice, be operating under the condition of $h < h_N$. This suggests that a partial remedy would be to use relatively shallow initial cells increasing them in depth as the pulp is depleted of its mineral content.

An arrangement of this nature might lead to considerable saving of power, without sacrifice in metallurgical results.

The Rate of Flotation and the Pulp-Density Factor -

To vary the pulp density in this series, 50 grams of galena were used in each test, but the amount of magnesite was increased in succeeding tests. The grade of the ore was therefore lowered as the pulp density increased, but the number of galena particles present in each test was the same. As the amount of magnesite was increased, the water was decreased so the pulp level was the same in the machine each time (measured at rest). All other conditions were the same as those previously described. The metallurgical results are tabulated below:

TABLE X.

Rate of Flotation and the Pulp Density Factor -

-150+200 Mesh Galena

Test No.	Solids in Pulp, %	Galena in Solids, %	Const. Wt., Gms.	Galena in Const., %	Galena in Const. Wt.	Frothing Time in Seconds	Galena Floated Per Min., Gms.
1	9.6	20.0	41.52	85.5	35.5	8.1	265
2	13.1	14.3	40.90	82.2	33.6	6.6	234?
3	18.0	10.0	44.45	80.2	35.6	7.8	274
4	21.0	8.34	45.19	75.8	34.3	7.4	278
5	25.5	6.66	49.26	66.2	32.5	7.8	250
6	32.2	5.00	54.89	57.7	31.6	7.0	271

The data of Table X show that, mineral concentration (number of particles per unit pulp volume) remaining constant, the pulp density as such has no marked effect on rate of recovery. The number of grams of galena floated per minute is variable in the several tests, it is true, but this variation does not indicate a clear trend. The inconsistency is, therefore, charged to experimental error. An error of one second in recording the frothing time is enough to account for the variation in grams floated per minute.

It is, however, interesting to note that the grade of the float is very noticeably affected by pulp density, i.e. the factor of ratio of tailing mineral particles to floatable mineral particles. When the pulp contains 9.6 per cent of solids, the concentrate contains 85.5 per cent of galena; whereas, when the pulp contains 32.2 per cent solids, the concentrate falls off in galena content to 57.7.

Referring again to "rate-of-flotation," in the light of previous researches in which the pulp density factor is shown to substantially affect aeration, it should be pointed out that there must have been a small falling off in aeration in the above series of tests, with increase of pulp consistency. The possible effect of this factor was not taken into account. It would be difficult to do so.

This experiment not only relates pulp density and rate of recovery, but it is, as well, a study of the relationship between ore-grade and rate-of-flotation.

* See Idaho Bureau of Mines and Geology Pamphlet No. 67.

Rate of Flotation and the Ore-Grade Factor -

Assume two ores, identical mineralogically, but one ore, A, containing x particles of mineral to be floated and $100 - x$ particles of gangue minerals, and another ore, B, containing $2x$ particles of mineral to be floated and $100 - 2x$ gangue particles. The above experimental data would seem to indicate that in a given volume of pulp mineral could be floated at the same rate from both ores providing pulp consistency were regulated to give the same number of floatable mineral particles per unit volume of pulp.

To further analyze the problem, let x have a specific gravity of 7.5 and let ore A contain 10 per cent of x and ore B 20 per cent of x . Let the specific gravity of the gangue mineral be 3.00. The pulp volumes to be floated are to be equal, but ore pulp A, if it is to contain the same percentage of mineral to be floated, is to contain twice as much ore by weight as pulp B. Now what will be the respective densities of the two pulps.

The specific gravities of the two ores are:

$$D_A = \frac{1.0}{\frac{0.10}{7.5} + \frac{0.90}{3.0}} = 3.20$$

$$D_B = \frac{1.0}{\frac{0.20}{7.5} + \frac{0.80}{3.0}} = 3.41$$

Now a feed pulp of ore A to contain as many floatable grains, in unit volume, as ore B will have to contain twice the weight of solids.

Assume for ore A a pulp of 40 per cent solids by weight. The density of ore A pulp would be

$$D_A = \frac{1.00}{\frac{0.40}{3.20} + \frac{0.60}{1.00}} = 1.38$$

The weight of 1000cc. of pulp then is $(1000 \times 1.38 =)$ 1380 grams. By definition, 40 per cent of this by weight is solids A, equal to 552 grams. Now, of these 552 grams, 10 per cent, or 55.2 grams are floatable mineral and $(552 - 55.2 =)$ 496.8 grams gangue. The volume of ore A in the pulp is $(552 \times \frac{1}{3.2} =)$ 172 cc.

Now 1000 cc. of pulp B also is to contain 55.2 grams of floatable mineral. The weight of ore B needed to provide this weight of floatable mineral is $(55.2 \times 20X =)$ 276 grams. The volume of ore B in the pulp is $(276 \times \frac{1}{3.41} =)$ 81 cc.

The volume of water in pulp B is $(1000 - 81 =)$ 919 cc., equal to 919 grams.

The density of pulp A, then, is $(\frac{276 + 919}{1000} =)$ 1.195 and the per cent solids is $(\frac{276}{276 + 1195} =)$ 23 per cent.

Pulp A, because of its higher density, will take more power per cubic cell content, but the theory is that the rate of flotation would be materially stepped up so that fewer cells would be required.

Low mineral content ores might well be floated at higher pulp consistencies (densities) than is the present practice. Fewer flotation machines would be one of the savings to result from such practice. Grade of concentrate, however, is a factor that cannot be overlooked.